Tapestry of Rays – The Misconceptions Series(1)

There are many misconceptions in the optical world. In this "Tapestry of Light – Misconceptions Series", some of these misconceptions will be taken up each time, in order to clarify them.

First, let us start with something very basic – the relationship between the F-number and NA (Numerical Aperture).

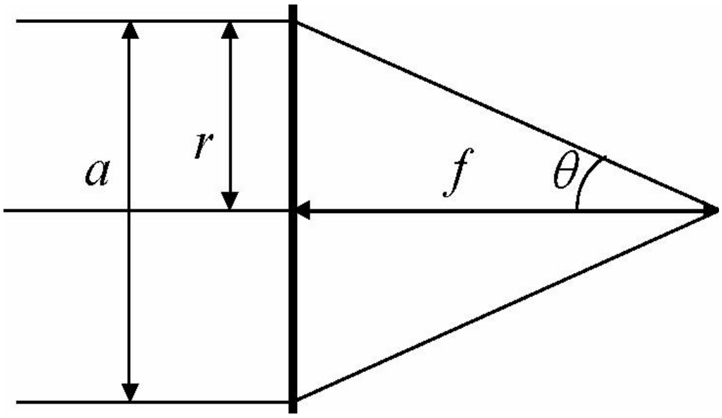
Both the F-number and NA are scales of the aperture of lens. They have the exact same meaning, and their conversion formula is surprisingly simple. It is:

$$F = \frac{1}{2NA} \qquad (1)$$

Those who are nodding in agreement with this formula need not read the following. However, some have a misconception that arcsin or arctan appears in this conversion formula, due to the following background.

First of all, the F-number is a number obtained by dividing focal length f by incident light flux a. This is illustrated in Figure 1.

Figure 1



Here, based on the definition:

$$F = \frac{f}{a} \qquad (2)$$

And from Figure 1:

$$\tan\theta = \frac{r}{f} = \frac{a}{2f} \qquad (3)$$

Therefore:

$$F = \frac{1}{2\tan\theta} \qquad (4)$$

Now, concerning NA:

$$NA = \sin\theta \qquad (5)$$

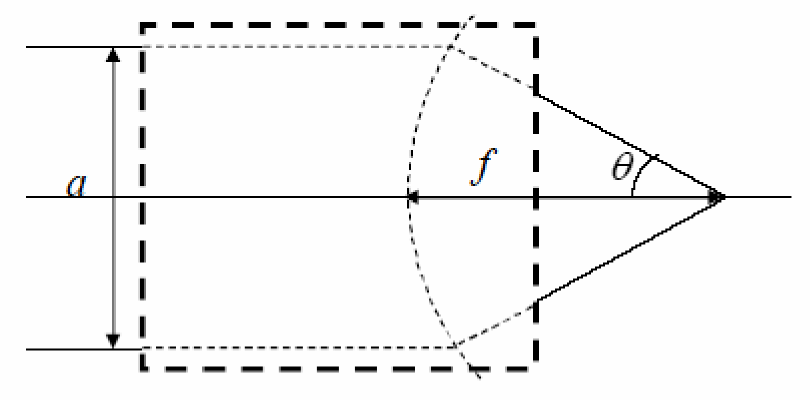
in the air, based on the definition. Obviously, (1) cannot possibly be derived from (4) and (5). Consequently, arcsin or arctan would appear. What goes wrong?

The biggest mistake is Figure 1.

In Figure 1, the lens is described as one planar surface. However, normal lenses should not be described this way. In good imaging, all the light rays relating to the imaging must possess equal optical path lengths between the object and the image. In the planar lens of Figure 1, the light rays passing through the circumference of the lens obviously have longer optical path lengths. In other words, the lens is forming an image geometrically; it is badly broken wave-optically.

In an actual lens, an image must also be formed wave-optically in a finite region. In order to realize this, the sine condition must be satisfied. Figure 2 is a picture of a lens that satisfies the sine condition. In Figure 2, the lens is described not as a planar surface, but as a black box surrounded by dotted lines. The dotted line arc in the figure indicates the outgoing wavefront of the light, and the light that exits the lens is forming a spherical wave converging on a single point. The distance between the incident face and the outgoing wavefront of the lens differ on the optical axis and on the circumference, as is clearly shown in the figure. However, the optical path lengths, obtained by multiplying the distance by the refractive index, become equal (are designed to become equal). For instance, a convex lens is thick at the center and thin at the circumference. Therefore, the light traveling on the optical axis has a long optical path length for the distance, and the opposite becomes true for the light traveling on the lens circumference.

Figure 2



The figure indicates the following:

$$F = \frac{a}{f} = \frac{1}{2\sin\theta}$$

(This is the sine condition itself). Therefore, formula (1) can be easily derived.

To be noted, there are exceptional cases among the existing optical system that can be described as Figure 1. They are grating lenses, such as Fresnel Zone Plate. Since these lenses do not satisfy the sine conditions, they can form images well only on the optical axis. We will not discuss general imaging concerning these lenses here, since such discussion is only meaningful in an exceptional configuration.